CHAPTER 2

Hydraulic Pumps and Motors

Hydraulic pumps and motors are the main elements of hydraulic drives and control systems. Their common function is energy conversion. Pumps convert mechanical energy into hydraulic energy which is stored as hydrostatic pressure of the working medium. Hydraulic motors, on the other hand, convert hydraulic energy into mechanical energy. It is important that such energy conversions are carried out with minimum energy losses, thus it is positive displacement pumps and motors that are principally used in hydraulic drives and control systems.

Positive displacement hydraulic pumps are characterised by separation of suction and pumping spaces by specially formed pumping elements whose movement displace fluid from suction to pumping spaces and thus force circulation of fluid in a system. The principle of operation of positive displacement hydraulic motors is inverse to that of pumps, fluid entering the motor forces movement of pumping elements, thus positive displacement pumps may operate as motors.

Positive displacement units, which are most commonly used in hydraulic control and drive systems, can be classified on the basis of form and arrangement of pumping elements into the following types:

- Gear pumps and motors - with internal or external gearing
- Piston pumps and motors - axial or radial
- Vane pumps and motors
- Rotary actuators
- Other units, not so commonly used, are screw pumps and motors, membrane pumps, etc.

A characteristic parameter of positive displacement units is stroke displacement q (the theoretical volume of fluid delivered by a pump or absorbed by a motor during one shaft revolution) or unit displacement V_ϕ (the theoretical volume of fluid delivered by a pump or absorbed by a motor per radian of shaft rotation). Displacement parameters q and V_ϕ are determined by the geometry of pumping elements. Using these parameters the theoretical flow rate Q_t of a pump or motor is:

$$Q_t = qn \qquad or \qquad Q_t = V_\phi \omega \tag{2.1}$$

where:

q	-	stroke displacement
n	-	shaft velocity
V_ϕ	-	unit displacement
ω	-	shaft angular velocity

Stroke and unit displacements are related by equation:

$$V_\phi = \frac{q}{2\pi} \tag{2.2}$$

Positive displacement pumps and motors may have either fixed or variable displacement. In the case of variable displacement units the theoretical flow rate Q_t is:

$$Q_t = \varepsilon q n \qquad or \qquad Q_t = \varepsilon V_\phi \omega \tag{2.3}$$

where ε is the displacement control parameter showing the ratio of a set to maximum displacement. The value of parameter ε varies $-1 \leq \varepsilon \leq 1$ for a pump and $0.2 \leq \varepsilon \leq 1$ for a motor. Theoretical power P_t of a pump or a motor is calculated from the relation:

$$P_t = Q_t \Delta p \tag{2.4}$$

and in the case of variable units:

$$P_t = \varepsilon q n \Delta p = \varepsilon V_\phi \omega \Delta p \tag{2.5}$$

Theoretical torque T_t of a pump or a motor, assuming steady-state operating conditions and absence of losses, can be derived from the equivalence of mechanical and hydraulic powers:

$$T_t \omega = Q_t \Delta p \tag{2.6}$$

where Δp is the difference of pressures at inlet and outlet ports. Theoretical torque is equal to:

$$T_t = \frac{Q_t}{\omega} \Delta p = V_\phi \Delta p \tag{2.7}$$

or

$$T_t = \frac{q}{2\pi} \Delta p \tag{2.8}$$

thus theoretical torque is dependent only on the unit displacement V_ϕ (or stroke displacement q_n) and the pressure differential Δp across of pump or motor. For variable units theoretical torque is equal to:

$$T_t = \frac{\varepsilon Q_t}{\omega}\ \Delta p = \varepsilon V_\phi \Delta p \tag{2.9}$$

or

$$T_t = \frac{\varepsilon q}{2\pi} \Delta p \tag{2.10}$$

The above relations are valid only for ideal units where losses which occur in actual pumps or motors have not been considered. Such losses are usually classified as volumetric losses and hydro-mechanical losses.

Volumetric losses reduce the theoretical flow rate of pumps or motors - they are mainly the result of internal leakages from high to low pressure zones in a pump or a motor. Volumetric losses are characterised by the volumetric efficiency of a unit.

Hydro-mechanical losses are the result of resistance to flow in internal passages of a pump or a motor and friction losses during the relative motion of moving parts. Hydro-mechanical losses manifest themselves as torque losses and are defined by the hydro-mechanical efficiency of a unit.

2.1 Positive displacement pumps

The purpose of positive displacement pumps is to supply a hydraulic system with fluid under a desired pressure - thus the pumps function as flow sources or flow generators.

The basic parameters which describe the operation of a pump are: theoretical delivery flow Q_t at a steady-state shaft rotation speed n_p and delivery pressure p_2, fig. 20.

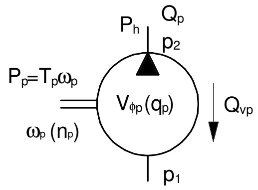

Fig. 20. Pump symbol

2.1.1 Volumetric efficiency

The theoretical delivery flow Q_{tp} of a pump is determined by its geometry and is not dependent on delivery pressure. The actual delivery flow Q_p of a pump is lower than the theoretical delivery flow Q_{tp} due to volumetric losses Q_{vp}, caused by leakage through clearances between the pumping elements and the pump body and by inadequate suction conditions (e.g. incomplete filling of suction chambers). Thus, if the value of volumetric losses are known, volumetric efficiency of the pump can be determined from the ratio of the actual flow rate Q_p to the theoretical flow rate Q_{tp}:

$$\eta_{vp} = \frac{Q_p}{Q_{tp}} = \frac{Q_{tp} - Q_{vp}}{Q_{tp}} = 1 - \frac{Q_{vp}}{q_p n_p} \tag{2.11}$$

From the above equation the actual delivery flow of a pump can be found when the theoretical delivery flow and the volumetric efficiency of a pump are known:

$$Q_p = Q_t \eta_{vp} = q_p n_p \eta_{vp} = V_{\phi p} \omega_p \eta_{vp} \tag{2.12}$$

As both leakage Q_{vp} and volumetric efficiency η_{vp} depend on pump pressure differential Δp_{12}, the volumetric losses which are due to the internal leakage can be calculated from equation:

$$Q_{vp} = K_{vp} \Delta p_{12} \tag{2.13}$$

where:
K_{vp} - pump leakage coefficient at a given fluid viscosity
Δp_{12} - pressure difference at inlet and outlet ports, $p_{12} \cong p_2$
thus the actual delivery flow of a pump is a function of the pressure. It should be stressed, that equation (2.13) is based on the assumption that internal leakage in a pump can be modelled as flow through close clearances (laminar flow). Taking into consideration the geometry of the clearances, the leakage flow rate can be calculated from the relation:

$$Q_{vp} = \frac{c^3 b}{12 l} \frac{\Delta p_{12}}{\mu} = K_{v\mu} V_{\phi p} \frac{\Delta p_{12}}{\mu} \tag{2.14}$$

where:
$K_{v\mu}$ - constant dependent on pump construction
l, b, c - length, width and height of the flow passage
μ - dynamic viscosity of the fluid.
Volumetric losses are proportional to the pressure difference and to the third power of the height of a clearance, thus they are proportional to clearances between mating surfaces of the elements which separate zones of different pressure.

Flow delivered by a pump will also be affected by incomplete filling of pumping chambers during suction (caused, for example, by aeration of the fluid or excessive pressure losses in a suction line), losses due to compressibility of the hydraulic fluid and deformation of pump elements. For correctly designed units these losses are small and more often than not may be ignored in further calculations. Thus, using equations (2.11) and (2.13) the following equations for volumetric efficiencies can be written:

$$\begin{aligned} &\text{for} \quad n_p = const. \quad \eta_{vp} = 1 - \frac{\Delta p_{12}}{const.} \\ &\text{for} \quad \Delta p_{12} = const. \quad \eta_{vp} = 1 - \frac{const.}{n_p} \end{aligned} \tag{2.15}$$

Typical plots of volumetric efficiency in function of pressure and rotation speed are shown in fig. 21.

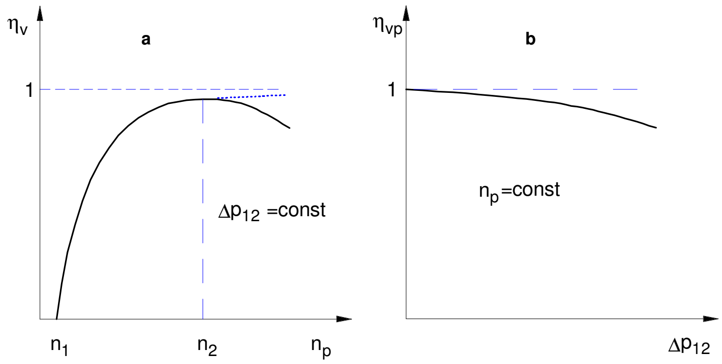

Fig. 21. Plot of volumetric efficiencies of a pump. a. η_{vp}=f(n_p), b. η_{vp}=f(Δp_{12})

Looking at the right hand side plot it can be seen that the volumetric efficiency decreases approximately linearly with increasing pressure. The actual changes in volumetric efficiency η_{vp} = f(Δp_{12}) are somewhat different from those shown in fig. 21b due to other volumetric losses mentioned above. It can also be noticed, that the graph shown in fig. 21b when plotted to a different scale will represent changes of the actual flow of the pump in function of the load, Q_p = f(Δp_{12}), [7].

The left hand side plot, fig. 21a, shows volumetric efficiency as a function of the rotation speed. When pump rotation speed is low $n < n_1$ then the volumetric efficiency $\eta_{vp} = 0$ as the pump delivery flow is not high enough to cover the internal leakage. As the speed of the pump is increased to above n_1 the volumetric efficiency quickly increases, reaches its maximum value at speed n_2, and then decreases due to the losses associated with incomplete filling of the pump during suction (cavitation).

2.1.2 Hydro-mechanical efficiency

Volumetric losses are accompanied by hydro-mechanical losses caused by fluid friction in pump passages (hydraulic losses) and by mechanical friction (mechanical

losses). The latter losses are represented as torque losses and are defined by hydro-mechanical efficiency η_{hmp}.
Hydro-mechanical efficiency of a positive displacement pump is expressed as the ratio of theoretical torque T_{tp} to actual torque T_p:

$$\eta_{hmp} = \frac{T_{tp}}{T_p} = \frac{T_{tp}}{T_{tp} + T_{fr}} = \frac{1}{1 + \dfrac{T_{fr}}{T_{tp}}} \tag{2.16}$$

where T_{tp} is theoretical (ideal) torque i.e. torque which would be required to drive the pump if there were no fluid or mechanical friction losses, eq. (2.7):

$$T_{tp} = V_{\phi p} \Delta p_{12} = \frac{q_p \Delta p_{12}}{2\pi} \tag{2.17}$$

and T_p is actual torque i.e. torque which is required to produce work. thus:

$$T_p = T_{tp} + T_{fr} \tag{2.18}$$

where:
T_{fr} - torque loss
Torque loss T_{fr} is the sum of torque losses:

$$T_{fr} = T_\mu + T_f + T_h + T_o \tag{2.19}$$

where:
T_μ - torque due to viscous friction(depends on speed and fluid viscosity)
T_f - torque due to mechanical friction which depends on load
T_h - torque due to hydraulic losses proportional to square of fluid velocity
T_o - a constant torque which is independent of motion
Typical plots of the actual torque for a positive displacement pump are shown in fig. 22.
In a modern positive displacement pump torque losses are mainly due to viscous friction T_μ (a function of speed and viscosity) and dry friction T_f (a function of load). According to [7] hydro-mechanical torque losses can be expressed by the following relations:

$$T_{fr} = \mathrm{f}(n) = A + Bn + Cn^2 \tag{2.20}$$

$$T_{fr} = \mathrm{f}(\Delta p) = D\Delta p + E \tag{2.21}$$

The above relations can be substituted in equation (2.16) to determine changes in the hydro-mechanical efficiency with varying speed or pressure, fig. 23.

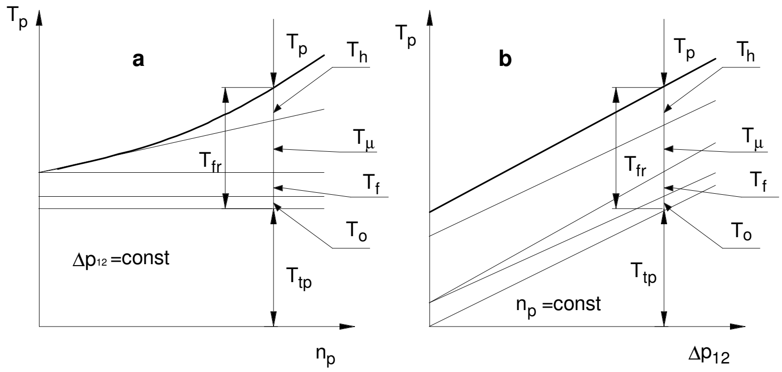

Fig. 22. Plots of actual pump torque T_p as a function of rotation speed n_p and pressure differential Δp_{12}

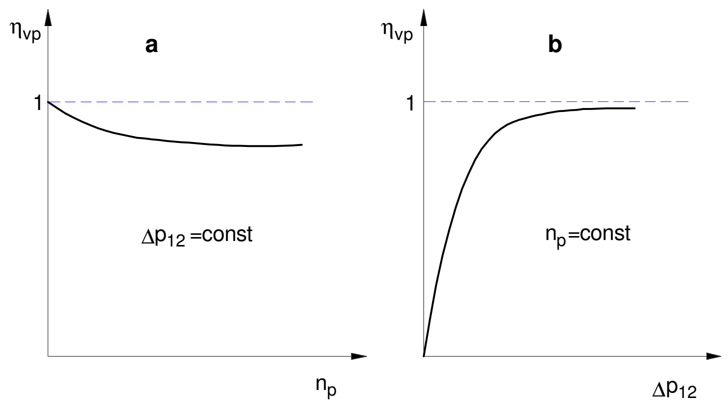

Fig. 23. Hydro-mechanical efficiency η_{hmp} of a pump as a function of: rotation speed n_p and pressure differential Δp_{12}

2.2 Positive displacement motors

Positive displacement hydraulic motors convert hydraulic energy into mechanical energy. Parameters which characterize hydraulic motors are: pressure differential Δp_{34} (pressure difference at inlet and return ports), actual demand flow of a motor Q_m and motor rotation speed n_m (or ω_m). These parameters are shown in fig. 24, the arrows illustrate the interdependence of the parameters; thus rotation speed of the motor is determined by its flow demand, and its delivery pressure is determined by the external load.

In the study of hydraulic motors we differentiate between a theoretical flow demand Q_{tm} and the actual flow demand Q_m. The theoretical (ideal) flow demand of a motor

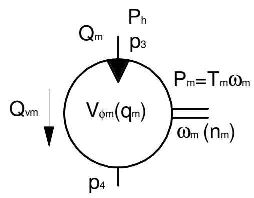

Fig. 24. Symbol of a positive displacement motor

is the volumetric flow rate through a motor with no volumetric losses. The presence of volumetric losses increases the theoretical flow demand of the motor to the actual flow demand. Thus, considering equation (2.1):

$$Q_m = Q_{tm} + Q_{vm} = \omega_m V_{\phi m} + Q_{vm} \tag{2.22}$$

It can also be assumed that, as in the case of pumps, the volumetric flow loss Q_{vm} is mainly dependent on the internal leakage and that it may be expressed by a relation analogical to that for a pump, eq. (2.13). Graphical interpretation of equation (2.22) is shown in fig. 25.

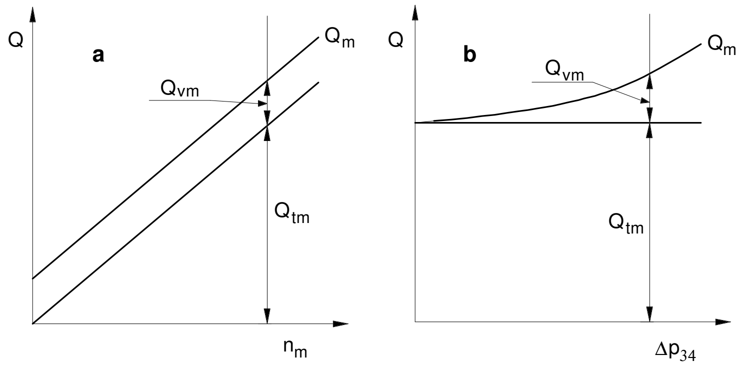

Fig. 25. Actual demand flow of a motor Q_m as a function of: rotation speed n_p and pressure differential Δp_{34}

2.2.1 Volumetric efficiency

Theoretical and actual flow demands of a hydraulic motor are related by volumetric efficiency η_{vm}. Volumetric efficiency of a motor is defined as the ratio of theoretical flow demand to actual flow demand, i.e.:

$$\eta_{vm} = \frac{Q_{tm}}{Q_m} = \frac{Q_{tm}}{Q_{tm} + Q_{vm}} \tag{2.23}$$

Using the above equation the output rotation speed of a motor can be determined:

$$n_m = \frac{Q_{tm}}{q_m} = \frac{Q_m}{q_m}\eta_{vm} \tag{2.24}$$

also the output angular speed can be defined by equation:

$$\omega_m = \frac{Q_m}{V_{\phi m}}\eta_{vm} \tag{2.25}$$

2.2.2 Hydro-mechanical efficiency

The second important output parameter in addition to shaft rotation speed is the actual output torque T_m. This torque is less than the theoretical torque T_{tm} due to, as discussed above, hydro-mechanical losses, eq. (2.19). Thus:

$$T_m = T_{tm} - T_{fr} \tag{2.26}$$

where T_{tm} is theoretical (ideal) torque derived from pressure differential across motor ports and the motor's geometry, eq. (2.7):

$$T_{tm} = V_{\phi m}\Delta p_{34} = \frac{q_m}{2\pi}\Delta p_{34} \tag{2.27}$$

and T_{fr} are torque losses caused by mechanical friction and hydraulic losses in the motor. Typical actual torque characteristics of a motor are shown in fig. 26.

The above equations show that for a motor to move, the actual differential pressure across motor ports must be higher than the differential pressure Δp_{34min}.

Hydro-mechanical efficiency of a motor η_{hmm} is defined as the ratio of the actual (input) torque to the theoretical (ideal) torque. Considering equations (2.26) and (2.27) the hydro-mechanical efficiency is defined by:

$$\eta_{hmm} = \frac{T_m}{T_{tm}} = \frac{T_{tm} - T_{fr}}{T_{tm}} = 1 - \frac{T_{fr}}{V_{\phi m}\Delta p_{34}} \tag{2.28}$$

Knowledge of hydro-mechanical efficiency and torque losses is very important as these parameters affect the starting characteristics of motors. It should also be added that, after the motor is stopped, start-up torque losses are considerably higher. This is due to a higher viscosity of fluid and the presence of mechanical static friction.

The value of output torque T_m can be calculated from relation (2.28) if the value of a hydro-mechanical efficiency η_{hmm} at given pressure differential Δp_{34} is known:

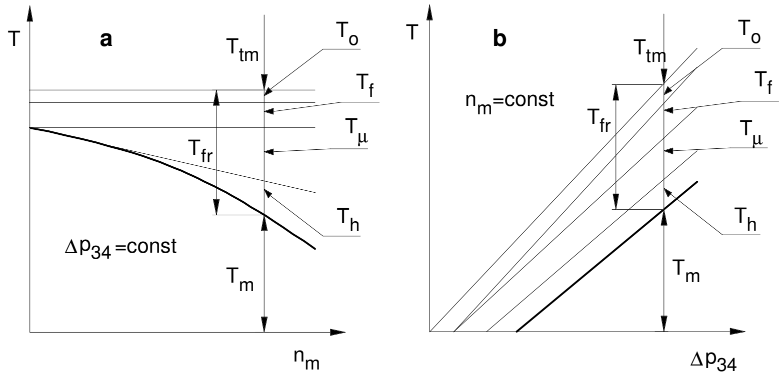

Fig. 26. Actual output torque T_m of a motor as a function of: rotation speed n_m and pressure differential Δp_{34}

$$T_m = T_{tm}\eta_{hmm} = V_{\phi m}\Delta p_{34}\eta_{hmm} = \frac{q_m}{2\pi}\Delta p_{34}\eta_{hmm} \tag{2.29}$$

From this equation the required value of delivery pressure can also be found when the load torque acting on the motor $T_{load} = T_m$ is known.

2.3 Overall efficiency of pumps and motors

2.3.1 Pumps

The hydraulic power of a pump, i.e. effective power which can be used by a hydraulic system is equal to the product of the actual delivery flow Q_p and the pressure differential Δp_{12}, thus:

$$P_h = Q_p\Delta p_{12} = Q_{tp}\eta_{vp}\Delta p_{12} = q_p n_p \eta_{vp}\Delta p_{12} \tag{2.30}$$

The input power, i.e. power which must be supplied by a prime mover, is equal to:

$$P_p = 2\pi n_p T_p = \omega_p T_p \tag{2.31}$$

The ratio of the effective power (hydraulic power) P_{hp} to the input power P_p describes the overall efficiency η_p of the pump:

$$\eta_p = \frac{P_{hp}}{P_p} = \frac{Q_p\Delta p_{12}}{T_p\omega_p} \tag{2.32}$$

and using equations (2.11), (2.13), (2.16) and (2.17) in equation (2.32) we find that the overall efficiency of a pump η_p is equal to the product of the volumetric efficiency η_{vp} and the hydro-mechanical efficiency η_{hmp}:

$$\eta_p = \eta_{vp}\eta_{hmp} \tag{2.33}$$

Characteristic parameters of pumps can be presented in the form of graphs which may subsequently be used as the basis for selection of a suitable pump. The following characteristics are most commonly used:

- plot of pump delivery as a function of pressure differential $Q_p = f(\Delta p)$ at $n = const.$
- plot of pump overall efficiency as a function of pressure differential $\eta_p = f(\Delta p)$ at $n = const.$
- plot of pump input power as a function of pressure differential $P_p = f(\Delta p)$ at $n = const.$, and
- plot of pump input power $P_p = f(n)$ or input torque $T_p = f(n)$ as functions of input speed at $\Delta p = const.$

A convenient form of presenting working parameters of a pump is a graph of universal characteristics which shows the plot of the pump flow rate Q_p as a function of pressure with superimposed plots of rotation speed, input power and efficiency. Such graphs allow quick selection of a pump suitable for a given hydraulic system.

2.3.2 Motors

Input and output parameters of a motor are related to the overall efficiency η_m of a motor. Overall efficiency is the ratio of the output mechanical power P_m at the motor shaft to the input hydraulic power P_{hm} supplied to the motor, where $P_{hm} = Q_m \Delta p_{34}$. Thus according to eq. (2.7):

$$\eta_m = \frac{P_m}{P_{hm}} = \frac{T_m \omega_m}{Q_m \Delta p_{34}} \tag{2.34}$$

and using equations (2.23) and (2.28) in equation (2.34), we can also write:

$$\eta_m = \eta_{vm}\eta_{hmm} \tag{2.35}$$

Volumetric, hydro-mechanical and overall efficiencies of a motor can be plotted as functions of the rotation speed n_m or as a function of pressure Δp_{34} (i.e. in a function of motor load). As in the case of pumps, working parameters of motors can also be shown using a graph of universal characteristics.

2.3.3 Dimensionless parameter σ_μ

A more convenient form of presenting changes of pump and motor characteristics is achieved by using a dimensionless parameter σ_μ defined as [5]:

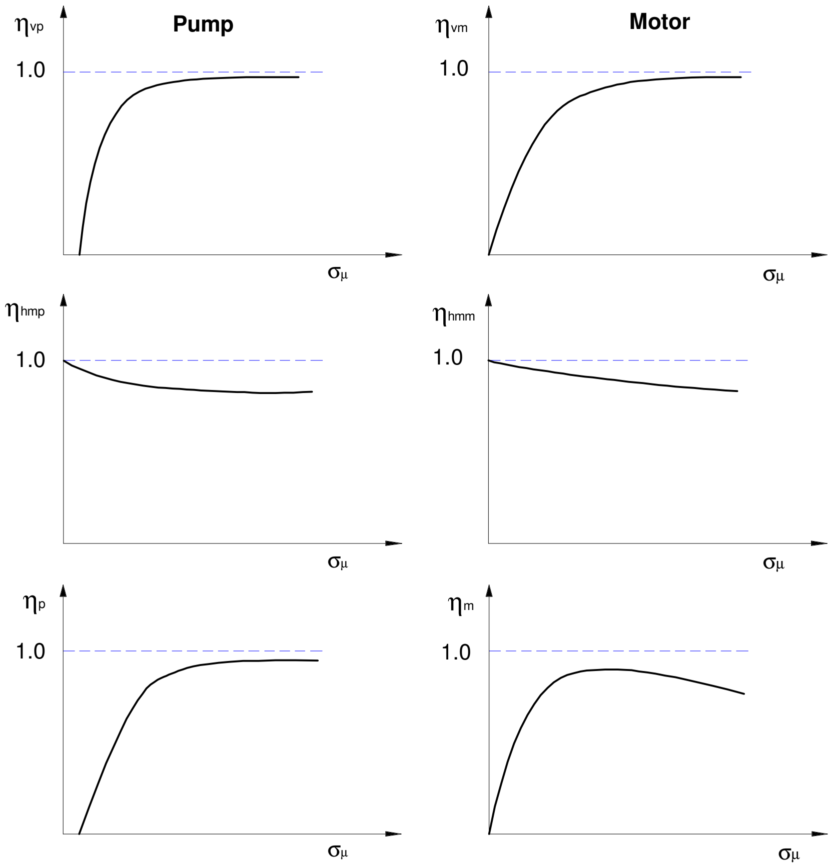

Fig. 27. Efficiencies of pump and motor as function of parameter σ_μ

$$\sigma_\mu = \frac{\omega\mu}{\Delta p} \tag{2.36}$$

This parameter is a general measure of the operating conditions of motors and pumps. Typical plots of volumetric, hydro-mechanical and overall efficiencies as functions of σ_μ for a pump (a) and a motor (b) are shown in fig. 27 on the previous page.

Examples of Calculations - Pumps and Motors

Problem 2.1 Theoretical and actual pump flow rate

Calculate theoretical delivery flow Q_{tp} and actual delivery flow Q_p of a pump which has stroke displacement $q_p = 5\ \text{cm}^3\text{rev}^{-1}$. The pump shaft rotation speed is $n_p = 2940$ rpm and the pump volumetric efficiency $\eta_{vp} = 0.9$.

Answer: The theoretical flow rate of the pump Q_{tp} is:

$$Q_{tp} = q_p n_p = 5.0 \times 10^{-6} \times 2940 = 14.7 \times 10^{-3}\ \text{m}^3\text{min}^{-1} = 14.7\ \text{Lmin}^{-1}$$

The actual flow rate Q_p is:

$$Q_p = Q_{tp}\eta_{vp} = 14.7 \times 0.9 = 13.23\ \text{Lmin}^{-1} \qquad \text{answer!}$$

Problem 2.2 Pump input torque

A pump which has unit displacement $q_p = 10\ \text{cm}^3\text{rev}^{-1}$ operates under pressure $p = 10$ MPa. What is the required input torque if pump hydro-mechanical efficiency is $\eta_{hmp} = 0.94$?

Answer: Hydro-mechanical efficiency of a pump is expressed by the relation:

$$\eta_{hmp} = \frac{T_{tp}}{T_p} = \frac{\Delta p_{12} V_{\phi p}}{T_p} = \frac{\Delta p_{12} q_p}{2\pi T_p}$$

thus:

$$T_p = \frac{\Delta p_{12} q_p}{2\pi \eta_{hmp}} = \frac{10 \times 10^6 \times 10 \times 10^{-6}}{2\pi \times 0.94} = 16.93\ \text{Nm} \qquad \text{answer!}$$

Problem 2.3 Delivery flow rate and volumetric efficiency of a pump

Calculate the actual delivery flow rate Q_p of a pump if its unit displacement is $V_\phi = 3\ \text{cm}^3\text{rad}^{-1}$and its shaft angular speed is $\omega_p = 300\,\text{rad}^{-1}$. Pump leakage loss is $Q_{vp} = 90\ \text{cm}^3\text{s}^{-1}$. What is the volumetric efficiency of the pump?

Answer: The theoretical delivery flow of the pump Q_{tp} is:

$$Q_{tp} = Q_p + Q_{vp}$$

and

$$Q_{tp} = V_\phi \omega_p = 3 \times 300 = 900\ \text{cm}^3\text{s}^{-1}$$

Thus actual delivery flow Q_p is:

$$Q_p = Q_{tp} - Q_{vp} = 900 - 90 = 810 \text{ cm}^3\text{s}^{-1}$$

and pump volumetric efficiency η_{vp} is equal to:

$$\eta_{vp} = \frac{Q_p}{Q_{tp}} = \frac{810}{900} = 0.9 \qquad \text{answer!}$$

Problem 2.4 Theoretical flow rate of a pump

Main dimensions of gears in a gear pump are:

gear width	$b = 30\,\text{mm}$
number of teeth	$z = 15$
modulus	$m = 3$

Calculate the theoretical delivery flow Q_{tp} when shaft speed is $n_p = 3000\,\text{rpm}$.

Answer: Stroke displacement of the pump is calculated using:

$$q_p = 2\pi m^2 zb = 25447\,\text{mm}^3\,\text{rev}^{-1}$$
$$q_p = 2.54 \times 10^{-5}\ \text{m}^3\text{rev}^{-1}$$

thus, the theoretical delivery flow Q_{tp} is:

$$Q_{tp} = n_p q_p = 3000 \times 2.54 \times 10^{-5} = 7.62 \times 10^{-2}\ \text{m}^3\text{min}^{-1} \qquad \text{answer!}$$

Problem 2.5 Actual flow rate and pressure of a pump

Assuming that, volumetric losses Q_{vp} in a pump depend only on the difference of pressures at pump inlet and outlet, develop an expression for the actual delivery flow Q_p of the pump when pressure difference between pressure and suction ports is Δp_{12}. The theoretical delivery flow of the pump is Q_{tp} and its volumetric efficiency is η_{vp} at nominal pressure p_n.

Answer: The actual flow delivery Q_p is:

$$Q_p = Q_{tp} - K_{vp}\Delta p_{12} \qquad \text{(a)}$$

As we know nominal pressure p_n and pump volumetric efficiency η_{vpn}, leakage coefficient K_{vp} can be calculated from equation:

$$Q_{pn} = Q_{tp} - K_{vp} p_n \tag{b}$$

and also

$$Q_{pn} = Q_{tp} \eta_{vpn} \tag{c}$$

where Q_{pn} is the actual delivery flow of the pump at nominal pressure p_n.
By comparing equations (b) and (c) we obtain:

$$Q_{tp} \eta_{vpn} = Q_{tp} - K_{vp} p_n$$

thus, the leakage coefficient K_{vp} is:

$$K_{vp} = \frac{Q_{tp} - Q_{tp} \eta_{vpn}}{p_n} = Q_{tp} \frac{1 - \eta_{vpn}}{p_n} \tag{d}$$

Finally, inserting (d) into (a) we obtain the expression:

$$Q_p = Q_{tp} \left(1 - \frac{1 - \eta_{vpn}}{p_n} \right) \Delta p_{12} \qquad \text{answer!}$$

which allows calculation of actual delivery flow Q_p at pressure difference Δp_{12} when the values of theoretical flow rate Q_{tp} and volumetric efficiency η_{vp} at some nominal pressure p_n are known.

Problem 2.6 Pump displacement, flow rate and power

Given the following data for a positive displacement pump:

- nominal pressure $p_n = 2.0 \times 10^7$ Pa
- nominal flow rate $Q_{pn} = 3.0 \times 10^{-2}$ m^{3}min^{-1}
- nominal rotation speed $n_n = 1440$ rpm
- volumetric efficiency $\eta_{vpn} = 0.95$
- hydro-mechanical efficiency $\eta_{hmpn} = 0.85$

Calculate:

- pump stroke displacement q_p and unit displacement $V_{\phi p}$
- theoretical delivery flow of the pump Q_{tp} and pump input power P_p
- volumetric efficiency η_{vp1} at pump speed $n_1 = 1000$ rpm (assume $Q_{vp} = \mathrm{f}(\Delta p)$)

Answer: The theoretical delivery flow of the pump is calculated using equation:

$$Q_{tp} = \frac{Q_{pn}}{\eta_{vpn}}$$

and

$$Q_{tp} = \frac{3 \times 10^{-2}}{60 \times 0.95} = 5.26 \times 10^{-4}\ \mathrm{m^3 s^{-1}} \qquad \text{answer!}$$

As effective pump power P_h is defined by equation:

$$P_h = Q_p \Delta p_{12} = Q_{tp} \eta_{vp} \Delta p_{12}$$

and pump input power P_p is equal to:

$$P_p = \frac{P_h}{\eta_{hmpn}}$$

then:

$$P_p = \frac{Q_{tp} \Delta p_{12}}{\eta_{hmpn}}$$

The pressure at pump inlet $p_1 \ll p_n$, so $\Delta p_{12} \approx p_n$, and pump power P_p at the nominal pressure p_n is:.

$$P_p = \frac{5.3 \times 10^{-4} \times 2.0 \times 10^7}{0.85} = 12.47\,\mathrm{kW} \qquad \text{answer!}$$

Pump stroke displacement q_p is defined by equation:

$$q_p = \frac{Q_{tp}}{n_n} = \frac{5.3 \times 10^{-4} \times 60}{1440} = 2.21 \times 10^{-5}\ \mathrm{cm^3 rev^{-1}} \qquad \text{answer!}$$

thus unit displacement $V_{\phi p}$ is:

$$V_{\phi p} = \frac{q_p}{2\pi} = \frac{2.21 \times 10^{-5}}{2\pi} = 3.52 \times 10^{-6}\ \mathrm{m^3 rad^{-1}} \qquad \text{answer!}$$

For nominal operating conditions the leakage coefficient K_{vp} is equal to:

$$K_{vp} = Q_{tp} \frac{1 - \eta_{vpn}}{p_n} = 5.3 \times 10^{-4} \frac{0.05}{2.0 \times 10^7} = 1.32 \times 10^{-12}\ \mathrm{N^{-1} m^5 s^{-1}}$$

and the theoretical delivery flow of the pump at rotation speed n_1 is equal to:

$$\begin{aligned} Q_{tp1} &= q_p n_1 = 2.21 \times 10^{-5} \times 1000 = 2.21 \times 10^{-2}\ \mathrm{m^3 min^{-1}} \\ Q_{tp1} &= 3.67 \times 10^{-4}\ \mathrm{m^3 s^{-1}} \end{aligned}$$

thus, the actual delivery flow Q_{p1} at rotation speed n_1 and pressure p_n is:

$$Q_{p1} = Q_{tp1} - K_{vp} p_n$$
$$Q_{p1} = 3.67 \times 10^{-4} - \left(1.3 \times 10^{-12} \times 2.0 \times 10^{7}\right) = 3.41 \times 10^{-4}\ \mathrm{m^3 s^{-1}}$$

Finally, the volumetric efficiency η_{vp1} at rotation speed $n_1 = 1000\,\mathrm{rpm}$ is equal to:

$$\eta_{vp1} = \frac{Q_{p1}}{Q_{tp1}} = \frac{3.41 \times 10^{-4}}{3.67 \times 10^{-4}} = 0.93 \qquad \text{answer!}$$

Problem 2.7 Efficiencies of a gear pump from test results

Calculate volumetric efficiency η_{vp}, hydro-mechanical efficiency η_{hmp} and overall efficiency η_p of a gear pump working under operating conditions listed in table 3. Coefficient of viscous losses $K_{v\mu} = 4 \times 10^{-8}$, and coefficient of viscous friction $K_{hm\mu} = 2 \times 10^{5}$.

Table 3. Test results - Problem 2.7

Test No.	**1**	**2**	**3**	**4**
Pressure Δp [MPa]	10	10	10	8
Dynamic viscosity μ [$\mathrm{Nsm^{-2}}$]	0.05	0.05	0.025	0.025
Rotation speed ω [$\mathrm{revs^{-1}}$]	10	20	20	20

Answer: If we assume, that volumetric losses and torque losses are due to viscosity of fluid, then leakage flow Q_{vp} can be calculated from eq.(2.14):

$$Q_{vp} = \frac{c^3 b \Delta p}{12 l \mu} \tag{a}$$

then using further assumption, that:

$$\frac{c^3 b}{12 l} = K_{v\mu} V_{\phi p}$$

(where coefficient $K_{v\mu}$ is a constant for a given type of pump) equation (a) becomes:

$$Q_{vp} = \frac{K_{v\mu} V_{\phi p} \Delta p}{\mu} = K_{vp} \Delta p \tag{b}$$

where:

$K_{v\mu}$, K_{vp} - coefficients of viscous losses and leakage
$V_{\phi p}$ - unit displacement of a pump
b, l, c - width, length and height of a clearance
μ - dynamic viscosity

Thus, if we substitute (b) in the expression for volumetric efficiency, we may write:

$$\eta_{vp} = \frac{Q_{tp} - Q_{vp}}{Q_{tp}} = 1 - \frac{K_{v\mu} V_{\phi p} \Delta p}{\omega V_{\phi p} \mu} = 1 - K_{v\mu} \frac{\Delta p}{\omega \mu} = 1 - \frac{K_{v\mu}}{\sigma_\mu} \tag{c}$$

where:

$$\sigma_\mu = \frac{\omega \mu}{\Delta p}$$

In a similar way, when determining the hydro-mechanical efficiency, we may assume that losses due to viscous friction have a dominating effect. These losses, as is well known, depend on angular speed ω and dynamic viscosity μ, thus we can define torque losses T_μ due to viscous friction as follows:

$$T_\mu = r \mu l b \frac{\omega r}{c} = \omega \mu \frac{l b r^2}{c}$$

and if we assume:

$$\frac{l b r^2}{c} = K_{hm\mu} V_{\phi p}$$

where:

$K_{hm\mu}$	-	coefficient of losses due to viscous friction
r	-	radius

we obtain:

$$\eta_{hmp} = \frac{T_{tp}}{T_{tp} + T_\mu} = \frac{V_{\phi p} \Delta p}{V_{\phi p} \Delta p + \omega \mu K_{hm\mu} V_{\phi p}}$$

and finally:

$$\eta_{hmp} = \frac{1}{1 + K_{hm\mu} \sigma_\mu} \tag{d}$$

Using equations (c) and (d) for the first series of tests:

$$\sigma_\mu = \frac{\omega \mu}{\Delta p} = \frac{2\pi \times 10 \times 0.05}{10^7} = 3.14 \times 10^{-7}$$

$$\eta_{vp} = 1 - \frac{K_{v\mu}}{\sigma_\mu} = 1 - \frac{4 \times 10^{-8}}{3.14 \times 10^{-7}} = 0.87 \qquad \text{answer!}$$

$$\eta_{hmp} = \frac{1}{1 + \sigma_\mu K_{hm\mu}} = \frac{1}{1 + 3.14 \times 10^{-7} \times 2 \times 10^5} = 0.94 \qquad \text{answer!}$$

$$\eta_p = \eta_{vp} \eta_{hmp} = 0.87 \times 0.94 = 0.82 \qquad \text{answer!}$$

For the second series of tests:

$$\begin{aligned}
\sigma_\mu &= \frac{\omega\mu}{\Delta p} = \frac{2\pi \times 20 \times 0.05}{10^7} = 6.28 \times 10^{-7} \\
\eta_{vp} &= 1 - \frac{K_{v\mu}}{\sigma_\mu} = 1 - \frac{4 \times 10^{-8}}{6.28 \times 10^{-7}} = 0.94 && \text{answer!} \\
\eta_{hmp} &= \frac{1}{1 + \sigma_\mu K_{hm\mu}} = \frac{1}{1 + 6.28 \times 10^{-7} \times 2 \times 10^5} = 0.89 && \text{answer!} \\
\eta_p &= \eta_{vp}\eta_{hmp} = 0.94 \times 0.89 = 0.84 && \text{answer!}
\end{aligned}$$

For the third series of tests:

$$\begin{aligned}
\sigma_\mu &= \frac{\omega\mu}{\Delta p} = \frac{2\pi \times 20 \times 0.025}{10^7} = 3.14 \times 10^{-7} \\
\eta_{vp} &= 1 - \frac{K_{v\mu}}{\sigma_\mu} = 1 - \frac{4 \times 10^{-8}}{3.14 \times 10^{-7}} = 0.87 && \text{answer!} \\
\eta_{hmp} &= \frac{1}{1 + \sigma_\mu K_{hm\mu}} = \frac{1}{1 + 3.14 \times 10^{-7} \times 2 \times 10^5} = 0.94 && \text{answer!} \\
\eta_p &= \eta_{vp}\eta_{hmp} = 0.87 \times 0.94 = 0.82 && \text{answer!}
\end{aligned}$$

And finally, for the fourth series of tests:

$$\begin{aligned}
\sigma_\mu &= \frac{\omega\mu}{\Delta p} = \frac{2\pi \times 20 \times 0.025}{8 \times 10^6} = 3.93 \times 10^{-7} \\
\eta_{vp} &= 1 - \frac{K_{v\mu}}{\sigma_\mu} = 1 - \frac{4 \times 10^{-8}}{3.93 \times 10^{-7}} = 0.90 && \text{answer!} \\
\eta_{hmp} &= \frac{1}{1 + \sigma_\mu K_{hm\mu}} = \frac{1}{1 + 3.93 \times 10^{-7} \times 2 \times 10^5} = 0.93 && \text{answer!} \\
\eta_p &= \eta_{vp}\eta_{hmp} = 0.90 \times 0.93 = 0.84 && \text{answer!}
\end{aligned}$$

Problem 2.8 Efficiencies of a gear pump from test results

The pump, driven with rotation speed $n_p = 1450\,\text{rpm}$ under load pressure $\Delta p_1 = 5$ MPa, has volumetric efficiency $\eta_{vp} = 0.9$ and overall efficiency $\eta_p = 0.85$. Dynamic viscosity of the fluid is $\mu_1 = 0.025$ Nsm^{-2}. According to catalogue data a gear pump has stroke displacement $q_p = 2 \times 10^{-6}$ m^{3}rev^{-1}.

Calculate volumetric η_{vp} and overall η_p efficiencies of the pump under the following operating conditions:

- pressure $\Delta p_2 = 7$ MPa,
- rotation speed $n_p = 950$ rpm
- fluid dynamic viscosity $\mu_2 = 0.05$ Nsm^{-2}.

Answer: Using catalogue data coefficient $\sigma_{\mu 1}$ is equal to:

$$\sigma_{\mu 1} = \frac{\omega_1 \mu_1}{\Delta p_1}$$
$$\sigma_{\mu 1} = \frac{2\pi \times 1450 \times 0.025}{5.0 \times 10^6 \times 60} = 7.59 \times 10^{-7}$$

Using equation (c) from Problem 2.7, the coefficient of viscous losses $K_{v\mu}$ is calculated from the following equation:

$$\eta_{vp1} = 1 - \frac{K_{v\mu}}{\sigma_{\mu 1}}$$

thus:

$$K_{v\mu} = \sigma_{\mu 1}(1 - \eta_{vp1}) = 7.59 \times 10^{-7}(1 - 0.9) = 7.59 \times 10^{-8}$$

The coefficient of friction losses $K_{hm\mu}$ due to viscous friction is calculated using eq. (d) from Problem 2.7:

$$\eta_{hmp1} = \frac{1}{1 + \sigma_{\mu 1} K_{hm\mu}}$$

and as

$$\eta_{hmp1} = \frac{\eta_p}{\eta_{vp}} = \frac{0.85}{0.9} = 0.944$$

thus:

$$K_{hm\mu} = \frac{1 - \eta_{hmp1}}{\sigma_{\mu 1}\eta_{hmp1}} = \frac{1 - 0.944}{7.59 \times 10^{-7} \times 0.944} = 7.82 \times 10^4$$

Since the values of coefficients $K_{v\mu}$ and $K_{hm\mu}$ are known, we may determine parameter $\sigma_{\mu 2}$:

$$\sigma_{\mu 2} = \frac{\omega_2 \mu_2}{\Delta p_2} = \frac{2\pi \times 950 \times .05}{70 \times 10^5 \times 60} = 7.10 \times 10^{-7}$$

thus

$$\eta_{vp2} = 1 - \frac{K_{v\mu}}{\sigma_{\mu 2}} = 1 - \frac{7.59 \times 10^{-8}}{7.1 \times 10^{-7}} = 0.89 \qquad \text{answer!}$$

$$\eta_{hmp2} = \frac{1}{1+\sigma_{\mu 2}K_{hm\mu}} = \frac{1}{1+7.10\times 10^{-7}\times 7.82\times 10^{4}} = 0.947 \quad \text{answer!}$$

Problem 2.9 Pump torque losses from results of tests

The results of tests on a hydraulic pump are shown in table 4. The pump has unit displacement $V_{\phi p} = 1.0$ cm^{3}rad^{-1}, dynamic viscosity of fluid $\mu = 0.02$ Nsm^{-2}and fluid density $\rho = 870\,\text{kg}\,\text{m}^{-3}$.

Table 4. Test results - Problem 2.9

Test No.	**1**	**2**	**3**	**4**
Pressure Δp [MPa]	0	5	10	10
Torque T_p [Nm]	0.46	5.56	10.91	10.44
Rotation speed ω_p [rads^{-1}]	200	200	300	100
Leakage flow Q_{vp} [m^{3}s^{-1}]	0	5×10^{-6}	10×10^{-6}	10×10^{-6}

Calculate on the basis of results obtained in above tests coefficients of the viscosity dependent volumetric loss $K_{v\mu}$ and the density dependent volumetric loss $K_{v\rho}$, using the following equation for leakage flow [7]:

$$Q_{vp} = \frac{K_{v\mu}V_{\phi p}\Delta p}{\mu} + K_{v\rho}\sqrt[3]{4\pi^2 V_{\phi p}^2}\sqrt{\frac{2\Delta p}{\rho}}$$

Calculate also torque losses T_{fr} using equation:

$$T_p = T_{tp} + T_{fr} \tag{a}$$

where T_{tp} is pump theoretical torque and torque loss T_{fr} is defined as in eq. (2.19):

$$\begin{aligned} T_{tp} &= V_{\phi p}\Delta p \\ T_{fr} &= T_\mu + T_f + T_h + T_o \end{aligned}$$

where:

- T_μ - torque due to viscous friction
- T_f - torque due to mechanical friction
- T_h - torque due to hydraulic losses
- T_o - a constant torque which is independent of motion

According to [7] torque losses T_μ, T_f and T_h are described by following equations:

$$\begin{aligned} T_\mu &= K_{hm\mu}\mu\omega_p V_{\phi p} \\ T_f &= K_{hmf}\Delta p V_{\phi p} \end{aligned}$$

$$T_h = K_{hmp} V_{\phi p}^{5/3} \frac{\rho\omega_p^2}{2}$$

where:

$K_{hm\mu}$	-	coefficient of viscous friction losses
K_{hmf}	-	coefficient of mechanical friction losses
$K_{hm\rho}$	-	coefficient of volumetric losses dependent on ρ

Finally equation (a) becomes:

$$T_p = V_{\phi p}\Delta p + K_{hm\mu}\mu\omega_p V_{\phi p} + K_{hmf}\Delta p V_{\phi p} + K_{hmp} V_{\phi p}^{5/3} \frac{\rho\omega_p^2}{2} + T_o \qquad \text{(b)}$$

Answer: As Q_{vp} is proportional to pressure Δp, thus volumetric losses are dependent only on fluid viscosity, thus:

$$K_{v\rho} = 0 \qquad \text{answer!}$$

and coefficients of the viscosity dependent volumetric loss $K_{v\mu}$ is equal to:

$$K_{v\mu} = \frac{Q_{vp}\mu}{\Delta p V_{\phi p}} = \frac{10 \times 10^{-6} \times 0.02}{10 \times 10^6 \times 1 \times 10^{-6}} = 2.0 \times 10^{-8} \qquad \text{answer!}$$

To calculate torque losses T_{fr} using equation (b) we need to find values of loss coefficients $K_{hm\mu}$, K_{hmf}, $K_{hm\rho}$ and constant torque loss T_0. Using results of test **1** and **2** we obtain:

$$0.46 = 0 + K_{hm\mu} \times 0.02 \times 200 \times 10^{-6} + 0 + +K_{hm\rho} \times 10^{-10} \times \frac{870 \times (200)^2}{2} + T_o \qquad \text{(c)}$$

$$5.56 = 5 \times 10^6 \times 1 \times 10^{-6} + K_{hm\mu} \times 0.02 \times 200 \times 10^{-6} + +K_{hmf} \times 5 \times 10^6 \times 10^{-6} + K_{hm\rho} \times 10^{-10} \times \frac{870 \times 200^2}{2} + T_o \qquad \text{(d)}$$

and subtracting by sides (c) from (d) we obtain:

$$0.10 = 5K_{hmf} \quad \text{therefore} \quad K_{hmf} = 0.02 \qquad \text{(e)}$$

From test **4** we obtain:

$$10.44 = 10 \times 10^6 \times 10^{-6} + K_{hm\mu} \times 0.02 \times 100 \times 10^{-6} + +K_{hmf} \times 10 \times 10^6 \times 10^{-6} + K_{hm\rho} \times 10^{-10} \times \frac{870 \times (200)^2}{2} + T_o \qquad \text{(f)}$$

subtracting both sides of (c) from (f) and using (e) we obtain:

$$-0.02 = -K_{hm\mu} \times 2 \times 10^{-6} + 0.2 - K_{hm\rho} \times 3 \times \frac{870}{2} \times 10^{-6} \tag{g}$$

From test **3** we have:

$$\begin{aligned} 10.91 &= 10 \times 10^6 \times 10^{-6} + K_{hm\mu} \times 0.02 \times 300 \times 10^{-6} + \\ &\quad +0.02 \times 10 \times 10^6 \times 10^{-6} + K_{hm\rho} \times 10^{-10} \times \frac{870 \times 300^2}{2} + T_0 \end{aligned} \tag{h}$$

again subtracting both sides of (c) from (h) we get:

$$0.45 = K_{hm\mu} \times 2 \times 10^{-6} + 0.2 + 5 \times K_{hm\rho} \times \frac{870}{2} \times 10^{-6} \tag{i}$$

and adding both sides of (i) and (g) we obtain:

$$0.43 = 0.4 + K_{hm\rho} \times 870 \times 10^{-6}$$

therefore:

$$K_{hm\rho} = 34.5 \tag{j}$$

We use equations (i) and (j) to obtain $K_{hm\mu}$:

$$0.45 = K_{hm\mu} \times 10^{-6} + 0.2 + 5 \times 34.5 \times \frac{870}{2} \times 10^{-6}$$

thus:

$$K_{hm\mu} = 8.75 \times 10^4 \tag{k}$$

To obtain T_0 we use (c), (j) and (k):

$$\begin{aligned} 0.46 &= 0 + 8.75 \times 10^4 \times 4 \times 10^{-6} + 0 + 34.5 \times 10^{-10} \times \frac{870}{2} \times 200^2 + T_0 \\ T_0 &= 0.05 \text{ Nm} \end{aligned}$$

Since values of coefficients $K_{hm\mu}$, K_{hmf}, $K_{hm\rho}$ and torque of constant loss $T_o = 0.05$ Nm are known we can determine, using eq. (b), torque losses in the pump for a given test condition. The results are summarised in table 5.

Table 5. Results of calculations - Problem 2.9

Torque loss [N]	**Test No.**			
	1	**2**	**3**	**4**
Viscous T_μ	0.35	0.35	0.52	0.18
Mechanical T_f	0	0.10	0.20	0.20
Hydraulic T_h	0.06	0.06	0.14	0.15
Constant T_0	0.05	0.05	0.05	0.05
Total T_p	0.46	0.57	0.91	0.58

Problem 2.10 Rotation speed and efficiency of a motor

A hydraulic motor with unit displacement $V_{\phi m} = 10 \times 10^{-6}$ m^{3}rad^{-1}has an actual flow demand $Q_m = 150 \times 10^{-6}$ m^{3}s^{-1}. Calculate:

- what will be the rotation speed of the motor when its volumetric efficiency $\eta_{vm} = 1.0$?
- what must be the actual flow demand Q_m to obtain the same rotation speed when volumetric efficiency $\eta_{vm} = 0.9$?

Answer: Theoretical flow demand is:

$$Q_{tm} = \omega_m V_{\phi m}$$

When volumetric efficiency of the motor is $\eta_{vm} = 1$, then $Q_m = Q_{tm}$, and:

$$\omega_m = \frac{Q_m}{V_{\phi m}} = \frac{150 \times 10^{-6}}{10 \times 10^{-6}} = 15.0 \text{ rads}^{-1}.$$

Rotation speed of the motor n_m is then:

$$n_m = \frac{\omega_m \times 60}{2\pi} = \frac{15 \times 60}{2\pi} = 143.24 \text{ revs}^{-1} \qquad \text{answer!}$$

The actual flow demand Q_m, when motor volumetric efficiency is $\eta_{vm} = 0.9$, can be calculated using expression:

$$\eta_{vm} = \frac{Q_{tm}}{Q_m} = \frac{\omega_m V_{\phi m}}{Q_m}$$

thus

$$Q_m = \frac{\omega_m V_{\phi m}}{\eta_{vm}}$$

$$Q_m = \frac{15 \times 10 \times 10^{-6}}{0.9} = 1.67 \times 10^{-4} \text{ m}^3\text{s}^{-1} \qquad \text{answer!}$$

Problem 2.11 Pressure difference across a motor

A hydraulic motor develops actual torque $T_m = 10$ Nm. What is the pressure difference Δp_{34} between supply and outlet ports of the motor when its hydro-mechanical efficiency $\eta_{hmm} = 0.95$ and its unit displacement $V_{\phi m}$=1×10^{-6} m^{3}rad^{-1}?

Answer: Hydro-mechanical efficiency is determined from the relation:

$$\eta_{hmm} = \frac{T_m}{T_{tm}} = \frac{T_m}{\Delta p_{34} V_{\phi m}}$$

thus the pressure difference Δp_{34} is:

$$\begin{aligned} \Delta p_{34} &= \frac{T_m}{\eta_{hmm} V_{\phi m}} \\ \Delta p_{34} &= \frac{10}{0.95 \times 1 \times 10^{-6}} = 10.5 \text{ MPa} \end{aligned} \qquad \text{answer!}$$

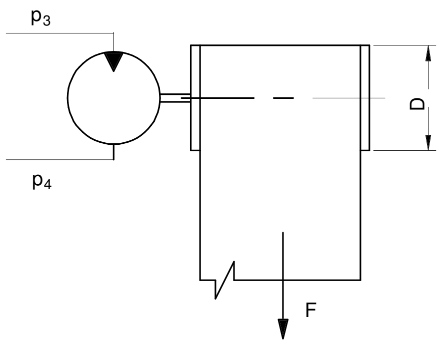

Fig. 28. Pulley drive

Problem 2.12 Size of hydraulic motor

A hydraulic motor drives a conveyor pulley, fig. 28. Maximum system pressure $p_3 = 7.0$ MPa. Calculate unit displacement $V_{\phi m}$ of the motor if the pulley has diameter $D = 260$ mm, belt tension force $F = 1000$ N and pressure in the return side of the motor $p_4 = 1.0$ MPa. Assume hydro-mechanical efficiency $\eta_{hmm} = 1$.

Answer: Unit displacement of the motor $V_{\phi m}$ is calculated from relation:

$$T_m = V_{\phi m} \Delta p_{34}$$

The torque loading T_{load} on the motor shaft is:

$$\begin{aligned} T_{load} &= F\frac{D}{2} \\ T_{load} &= 1000 \times \frac{0.260}{2} = 130 \text{ Nm} \end{aligned}$$

The pressure difference Δp_{34} between supply and return ports of the motor is:

$$\Delta p_{34} = 7.0 - 1.0 = 6.0 \text{ MPa}$$

and as:

$$T_m = T_{load}$$

Thus required unit displacement $V_{\phi m}$ of the motor is:

$$\begin{aligned} V_{\phi m} &= \frac{T_{load}}{\Delta p_{34}} \\ V_{\phi m} &= \frac{130.0}{6.0 \times 10^6} = 2.17 \times 10^{-5} \text{ m}^3\text{rad}^{-1} \end{aligned}$$ answer!

Problem 2.13 Theoretical torque of a radial piston motor

A 3-cylinder radial piston motor has the following geometric parameters:

piston diameter	$d = 11\,\text{mm}$
eccentricity	$e = 8\,\text{mm}$
number of pistons	$z = 11$
number of cylinders	$a = 3$

Calculate:

- stroke displacement q_m
- unit displacement $V_{\phi m}$ and
- theoretical torque T_{tm} when pressure differential across the motor $\Delta p_{34} = 25$ MPa.

Answer: Stroke displacement of a radial piston motor is calculated using the following equation:

$$q_m = \frac{\pi d^2}{2} eza$$

thus:

$$q_m = \frac{\pi (11 \times 10^{-3})^2}{2} \times 8 \times 10^{-3} \times 11 \times 3 = 5.0 \times 10^{-5} \text{ m}^3\text{rev}^{-1}$$ answer!

and the motor unit displacement is:

$$V_{\phi m} = \frac{q_m}{2\pi}$$
$$V_{\phi m} = \frac{5.0 \times 10^{-5}}{2\pi} = 7.96 \times 10^{-6}\ \mathrm{m^3rad^{-1}} \qquad \text{answer!}$$

Theoretical torque of the motor:

$$T_{tm} = V_{\phi m} \Delta p_{34}$$
$$T_{tm} = 7.96 \times 10^{-6} \times 25 \times 10^{6} = 199.0\ \mathrm{Nm} \qquad \text{answer!}$$

Problem 2.14 Leakage flow in a hydraulic motor

What is the leakage flow Q_{vm} of a motor which has actual flow demand $Q_m = 60\ \mathrm{Lmin^{-1}}$, its unit displacement $V_{\phi m} = 8 \times 10^{-6}\ \mathrm{m^3rad^{-1}}$and rotation speed $\omega_m = 120\ \mathrm{rads^{-1}}$?

Answer: Actual flow demand Q_m of the motor is:

$$Q_m = Q_{tm} + Q_{vm}$$

and as theoretical flow demand Q_{tm} is equal to:

$$Q_{tm} = V_{\phi m} \omega_m$$
$$Q_{tm} = 8 \times 10^{-6} \times 120 = 9.6 \times 10^{-4}\ \mathrm{m^3s^{-1}}$$

then, leakage flow Q_{vm}:

$$Q_{vm} = Q_m - Q_{tm}$$
$$Q_{vm} = \frac{60 \times 10^{-3}}{60} - 9.6 \times 10^{-4} = 4.0 \times 10^{-5}\ \mathrm{m^3s^{-1}} \qquad \text{answer!}$$

Problem 2.15 Hydraulic motor - actual torque and torque losses

A hydraulic motor which has unit displacement $V_{\phi m} = 10.5 \times 10^{-6}\ \mathrm{m^3rad^{-1}}$operates under the following conditions:

- inlet pressure $p_3 = 10$ MPa
- outlet pressure $p_4 = 0.2$ MPa.
- actual output torque is $T_m = 100$ Nm.

Calculate torque losses T_{fr} under the above operating conditions.

Answer: Pressure differential which generates the torque is:

$$p_{34} = p_3 - p_4$$
$$p_{34} = 10 - 0.2 = 9.8 \text{ MPa}$$

and torque losses are calculated from equation:

$$T_{fr} = T_{tm} - T_m = V_{\phi m} \Delta p_{34} - T_m$$
$$T_{fr} = 10.5 \times 10^{-6} \times 9.8 \times 10^6 - 100 = 2.9 \text{ Nm} \qquad \text{answer!}$$

Problem 2.16 Pump flow and system pressure

A simplified diagram of a hydraulically driven transfer car is shown in fig. 29 where unit displacements of the motors are $V_{\phi m1} = 75 \times 10^{-6}$ m^{3}rad^{-1} and $V_{\phi m2} = 150 \times 10^{-6}$ m^{3}rad^{-1}. Radii of wheels are $r_1 = 0.5$ m and $r_2 = 0.75$ m. Assume no losses in the system.
Calculate:

- pressure p_3 in the system if the required driving force $F_c = 10^4$ N.
- actual (required) delivery flow Q_p of the pump if required speed of transfer car is v = 18 km per hour.

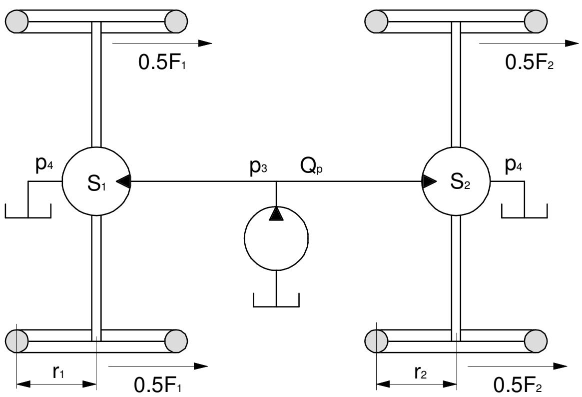

Fig. 29. Drive of a transfer car

Answer: Torque T_{m1} of motor S_1:

$$T_{m1} = V_{\phi m1} \Delta p_{34} = 2r_1 \frac{F_1}{2}$$

from the above driving force F_1 is:

$$F_1 = \frac{V_{\phi m1}}{r_1}\Delta p_{34}$$

Torque T_{m2} of motor S_2:

$$T_{m2} = V_{\phi m2}\Delta p_{34} = 2r_2\frac{F_2}{2}$$

and driving force F_2 is equal to:

$$F_2 = \frac{V_{\phi m2}}{r_2}\Delta p_{34}$$

Total driving force F_c:

$$F_c = F_1 + F_2$$

We can assume that return line pressure p_4 is small, thus $\Delta p_{34} \approx p_3$, and delivery pressure p_3 can be calculated from:

$$p_3 = \frac{F_c}{\frac{V_{\phi m1}}{r_1} + \frac{V_{\phi m}}{r_2}}$$

$$p_3 = \frac{10000}{\frac{75.0 \times 10^{-6}}{0.5} + \frac{150.0 \times 10^{-6}}{0.75}} = 28.6\,\text{MPa} \qquad \text{answer!}$$

The actual pump flow rate must be equal to the total flow demand of motors, thus:

$$Q_p = Q_{m1} + Q_{m2}$$

Angular velocities of motors are calculated from the following relations (transfer car speed $v = 5\,\text{ms}^{-1}$):

$$\omega_1 = \frac{v}{r_1}$$

$$\omega_1 = \frac{5.0}{0.5} = 10.0 \text{ rads}^{-1}$$

$$\omega_2 = \frac{v}{r_2}$$

$$\omega_2 = \frac{5.0}{0.75} = 6.67 \text{ rads}^{-1}$$

The actual flow demand of motor S_1 is:

$$
\begin{aligned}
Q_{m1} &= V_{\phi m}\omega_1 \\
Q_{m1} &= 75.0 \times 10^{-6} \times 10.0 = 0.750 \times 10^{-3} \text{ m}^3\text{s}^{-1}
\end{aligned}
$$

and the demand flow of motor S_2 is equal to:

$$
\begin{aligned}
Q_{m2} &= V_{\phi m}\omega_2 \\
Q_{m2} &= 150.0 \times 10^{-6} \times 6.77 = 1.02 \times 10^{-3} \text{ m}^3\text{s}^{-1}
\end{aligned}
$$

Finally, the required pump delivery flow Q_p is:

$$
\begin{aligned}
Q_p &= Q_{m1} + Q_{m2} \\
Q_p &= (0.75 + 1.02) \times 10^{-6} = 1.77 \times 10^{-6} \text{ m}^3\text{s}^{-1} \qquad \text{answer!}
\end{aligned}
$$

Problem 2.17 Pump working as a motor

Some positive displacement pumps are designed to be used also as motors. Working as a pump, the operating parameters are:

nominal pressure	$p_n = 2 \times 10^7$ Pa
nominal flow delivery	$Q_{pn} = 3 \times 10^{-2}$ m^{3}min^{-1}
nominal rotation speed	$n_n = 1440$ rpm
volumetric efficiency	$\eta_{vp} = 0.95$
hydro-mechanical efficiency	$\eta_{hmp} = 0.85$

Calculate:

- Theoretical T_{tm} and actual T_m output torque when the unit works as a motor
- Theoretical P_{tm} and actual power P_m when the unit works as a motor

Answer: As we know the operating parameters of the unit when it operates as a pump we can find its parameters when it is used as a motor. We also know that the volumetric efficiency of a motor is:

$$\eta_{vm} = \frac{Q_{tm}}{Q_m}$$

thus when the volumetric efficiency of the unit when it works as a pump is known we can calculate its volumetric efficiency when it works as a motor using relation:

$$\eta_{vm} = \frac{1}{2 - \eta_{vp}}$$

Hydro-mechanical efficiency of a motor is defined by the expression:

$$\eta_{hmm} = \frac{T_m}{T_{tm}}$$

and it can be also calculated using the hydro-mechanical efficiency of the unit when it works as a pump:

$$\eta_{hmm} = 2 - \frac{1}{\eta_{hmp}}$$

The above equations show that the efficiencies of the units are different depending on whether the unit works as a pump or as a motor. Assuming that the pump theoretical delivery is equal to the motor theoretical demand:

$$Q_{tp} = Q_{tm}$$

and also:

$$\begin{aligned} Q_{tp} &= \frac{Q_{pn}}{\eta_{vp}} \\ Q_{tm} &= Q_{mn}\eta_{vm} \end{aligned}$$

we obtain nominal and theoretical flow demands of the motor:

$$\begin{aligned} Q_{mn} &= \frac{Q_{pn}}{\eta_{vm}\eta_{vp}} \\ Q_{tm} &= \frac{Q_{pn}}{\eta_{vp}} \end{aligned}$$

Then, as unit volumetric efficiency when working as a pump is $\eta_{vp} = 0.95$, unit efficiency working as a motor is:

$$\begin{aligned} \eta_{vm} &= \frac{1}{2 - \eta_{vp}} \\ \eta_{vm} &= \frac{1}{2 - 0.95} = 0.95 \end{aligned}$$

The nominal flow demand of the motor Q_{mn} is equal to:

$$Q_{mn} = \frac{3 \times 10^{-2}}{0.95 \times 0.95} = 3.32 \times 10^{-2}\ \mathrm{m^3 min^{-1}}$$

and theoretical flow demand Q_{tm} is:

$$Q_{tm} = 3.32 \times 10^{-2} \times 0.95 = 3.15 \times 10^{-2}\ \mathrm{m^3 min^{-1}}$$

Theoretical torque T_{tm} and actual torque T_m for nominal conditions are:

$$
\begin{aligned}
T_{tm} &= \frac{Q_{tm}}{2\pi n_n}\Delta p_n \\
T_{tm} &= \frac{3.15\times 10^{-2}}{2\pi\times 1440} 2\times 10^7 = 69.6\ \text{Nm} \qquad \text{answer!}
\end{aligned}
$$

and as

$$T_m = T_{tm}\eta_{hmm}$$

where:

$$
\begin{aligned}
\eta_{hmm} &= 2 - \frac{1}{\eta_{hmp}} \\
\eta_{hmm} &= 2 - \frac{1}{0.85} = 0.82
\end{aligned}
$$

then

$$T_m = 69.6\times 0.82 = 57.0\ \text{Nm} \qquad \text{answer!}$$

Theoretical power of the unit operating as a motor at nominal operating conditions is calculated using equation:

$$
\begin{aligned}
P_{tm} &= T_{tm}\omega_n = V_{\phi m} p_n \omega_n = Q_{tm} p_n \\
P_{tm} &= P_{tp} \\
P_{tm} &= \frac{3.15\times 10^{-2}}{60} 2\times 10^7 = 10.5\,\text{kW} \qquad \text{answer!}
\end{aligned}
$$

and finally actual power of the motor:

$$
\begin{aligned}
P_m &= P_{tm}\eta_{hmm} \\
P_m &= 10.7\times 0.82 = 8.8\,\text{kW} \qquad \text{answer!}
\end{aligned}
$$

Problem 2.18 Rotation speed and efficiency of a motor

The known operating parameters of a motor are pressure differential $\Delta p_m = p_a$, volumetric efficiency η_{vma} and rotation speed n_a. What is the expression for rotation speed n_b of the motor if the load pressure $\Delta p_m = p_b$ and its volumetric efficiency is η_{vmb}?

Answer: Rotation speed of the motor can be calculated if we know the theoretical flow demand of the motor and its stroke displacement q_m or the actual flow demand and volumetric efficiency η_{vm}:

$$n_m = \frac{Q_{tm}}{q_m} = \frac{Q_m \eta_{vm}}{q_m}$$

The actual flow demand Q_m:

$$Q_m = Q_{tm} + K_{vm}\Delta p_m$$

or

$$Q_m = Q_m \eta_{vm} + K_{vm}\Delta p_m$$

thus motor leakage coefficient:

$$K_{vm} = \frac{Q_m(1 - \eta_{vm})}{\Delta p_m}$$

Using values of volumetric efficiency $\eta_{ma} = \eta_{vma}$ and pressure differential $\Delta p_a = p_a$ expression for coefficient K_{vm} becomes:

$$K_{vm} = \frac{Q_{ma}(1 - \eta_{vma})}{p_a}$$

and as

$$Q_{ma} = \frac{n_a q_m}{\eta_{vma}}$$

then

$$K_{vm} = q_m \frac{n_a}{p_a} \frac{1 - \eta_{vma}}{\eta_{vma}}$$

or

$$K_{vm} = q_m \frac{n_a}{p_a} \left(\frac{1}{\eta_{vma}} - 1 \right)$$

Value of K_{vm} can be used to calculate rotation speed n_b if we assume that coefficient K_{vm} has a constant value for a given type of hydraulic motor. The rotation speed of the motor, when $\Delta p_m = p_b$ and the value of the coefficient of volumetric efficiency η_{vb}, is calculated in a similar way:

$$n_b = \frac{Q_{tmb}}{q_m} = \frac{Q_{mb}}{q_m}\eta_{vmb}$$

thus

$$Q_{mb} = Q_{tmb} + K_{vm}p_b$$

Replacing Q_{mb} with $Q_{mb} = \dfrac{Q_{tmb}}{q_m}$ and substituting for K_{vm} we obtain:

$$\frac{Q_{tmb}}{\eta_{vmb}} = Q_{tmb} + q_m\frac{n_a}{p_a}(\frac{1}{\eta_{vma}} - 1)p_b$$

Dividing both sides of above equation by $q_m = const.$ and rearranging we get:

$$\frac{Q_{tmb}}{q_m}(\frac{1}{\eta_{vmb}} - 1) = \frac{n_a p_b}{p_a}\left(\frac{1}{\eta_{vma}} - 1\right)$$

If we substitute

$$n_b = \frac{Q_{tmb}}{q_m}$$

into the above equation, then

$$n_b(\frac{1}{\eta_{vmb}} - 1) = \frac{n_a p_b}{p_a}(\frac{1}{\eta_{vma}} - 1)$$

and finally rotation speed n_b is

$$n_b = \frac{n_a p_b}{p_a}\frac{(\frac{1}{\eta_{vma}} - 1)}{(\frac{1}{\eta_{vmb}} - 1)}$$

or

$$n_b = \frac{n_a p_b \eta_{vma}}{p_a \eta_{vmb}}\frac{(1 - \eta_{vma})}{(1 - \eta_{vmb})} \qquad \text{answer!}$$

Problem 2.19 Rotation speed of a motor

For a hydraulic unit in Problem 2.17, working as a motor, calculate:

- theoretical n_{tm} and actual n_m rotation speeds if the delivery flow is $Q_{m2} = 1.5 \times 10^{-2}\ \mathrm{m^3 min^{-1}}$and pressure differential is equal to $\Delta p_m = p_3 - p_4 = 10^7\,\mathrm{Pa}$.
- actual rotation speed n_m when pressure differential $\Delta p_m = 2 \times 10^7\,\mathrm{Pa}$.

Answer: The unit displacement of the motor:

$$q_m = q_p = \frac{Q_{pn}}{n_n \eta_{vp}}$$
$$q_m = \frac{3 \times 10^{-2}}{1440 \times 0.95} = 2.19 \times 10^{-5}\ \mathrm{m^3 rev^{-1}}$$

The theoretical rotation speed when volumetric efficiency of the motor is $\eta_{vm} = 1$:

$$n_{tm} = \frac{Q_m}{q_m}$$
$$n_{tm} = \frac{1.5 \times 10^{-2}}{2.19 \times 10^{-5}} = 685\ \mathrm{m^3 min^{-1}} \qquad \text{(answer!)}$$

The actual rotation speed of a motor is defined by the relation:

$$n_m = \frac{Q_{tm}}{q_m} = \frac{Q_m - K_{vm}\Delta p_m}{q_m}$$
$$n_m = n_{tm} - \frac{K_{vm}\Delta p_m}{q_m}$$

To calculate leakage coefficient K_{vm} for the motor we can use equation:

$$K_{vm} = \frac{Q_m}{\Delta p_m}(1 - \eta_{vm})$$

The theoretical flow demand of the motor (see calculations in Problem 2.17) is:

$$Q_{tm} = 3.15\ \times 10^{-2}\ \mathrm{m^3 min^{-1}}$$

thus, when motor volumetric efficiency is $\eta_{vm} = 0.95$ flow demand of the motor is equal to:

$$Q_m = \frac{Q_{tm}}{\eta_{vm}}$$
$$Q_m = \frac{3.15\ \times 10^{-2}}{0.95} = 3.32 \times 10^{-2}\ \mathrm{m^3 min^{-1}}$$

and coefficient K_{vm} for nominal conditions is then:

$$K_{vm} = \frac{3.32 \times 10^{-2}}{2 \times 10^{7}} \times (1 - 0.95) = 8.32 \times 10^{-11}\ \mathrm{N^{-1} m^5 min^{-1}}$$

At a constant fluid temperature value K_{vm} can be assumed to be constant, thus the rotation speed is defined by:

$$n_m = 685 - \frac{8.32 \times 10^{-11}}{2.19 \times 10^{-5}} \times \Delta p_m$$
$$n_m = 685 - 3.8 \times 10^{-6} \Delta p_m$$

This is a general equation for rotation speed of this hydraulic motor which depends on pressure differential across the motor which itself is a function of the external load on the shaft. For pressure $\Delta p_m = p_3 = 10^7$ Pa and motor flow demand $Q_m = 1.5 \times 10^{-2}$ $m^3 min^{-1}$the rotation speed is:

$$n_m = 685 - 3.8 \times 10^{-6} \times 10^7 = 647.0 \text{ rpm} \qquad \text{answer!}$$

In the case when the motor supply pressure is doubled, $\Delta p_m = 2 \times 10^7$ Pa and the actual flow demand of the motor is $Q_m = 1.5 \times 10^{-2}$ $m^3 min^{-1}$, the motor rotation speed is:

$$n_m = 685 - 3.8 \times 10^{-6} \times 2 \times 10^7 = 609.0 \text{ rpm} \qquad \text{answer!}$$

Problem 2.20 Hydro-mechanical efficiency of a motor

Table 6 shows the results of tests on a hydraulic motor. The objective of the tests was to determine the hydro-mechanical efficiency of the motor. Calculate the components of the torque loss T_{loss} for the case when:

pressure differential	$\Delta p = 10$ MPa
angular velocity	$\omega = 100$ $rads^{-1}$
dynamic viscosity	$\mu = 0.05$ Nsm^{-2}

Table 6. Experimental data - Problem 2.20

Test No	**1**	**2**	**3**	**4**	**5**	**6**
Pressure Δp [MPa]	10	10	10	10	10	20
Torque T_{loss} [Nm]	8	14	32	22	44	13
Rotation speed ω $rads^{-1}$	0	50	100	100	200	0
Viscosity μ [Nsm^{-2}]	0.05	0.05	0.10	0.05	0.05	0.05

Answer: The results of tests are plotted in fig. 30-32. Graph in fig. 30 shows $T_{loss} = f(\Delta p)$, graph in fig. 31 shows $T_{loss} = f(\omega)$ and graph in fig. 32 $T_{loss} = f(\mu)$.

The equations for torque losses are (see Problem 2.9):

$$T_\mu = K_{hm\mu} \mu \omega V_{\phi m}$$

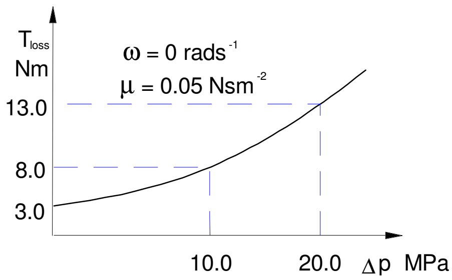

Fig. 30. Motor torque losses as function of pressure, $T_{loss} = f(\Delta p)$

$$\begin{aligned} T_f &= K_{hmf}\Delta p V_{\phi m} \\ T_h &= K_{hm\rho} V_{\phi m}^{5/3} \frac{\rho\omega^2}{2} \\ T_0 &= const. \end{aligned}$$

and

$$T_{loss} = T_\mu + T_f + T_h + T_0$$

We can see in fig. 30 that for $\mu = 0.05$, $\omega = 0$ and $p = 0$ torque losses are $T_{loss} =$ 3 Nm, and as for $p = 0$ and $\omega = 0$ losses $T_\mu = T_f = T_h = 0$ then:

$$T_0 = T_{loss} = 3 \text{ Nm} \qquad \text{answer!}$$

We use again fig. 30 to determine mechanical torque losses T_f. Torque losses T_h and T_μ are zero when angular velocity $\omega = 0$, however for $\Delta p = 10$ MPa torque losses are $T_{loss} = T_f + T_o = 8$ Nm thus:

$$T_f = T_{loss} - T_0 = 5 \text{ Nm} \qquad \text{(answer!)}$$

Fig. 31 shows plot of torque losses $T_{loss} = f(\omega)$ at constant pressure differential $\Delta p = 10$ MPa and viscosity $\mu = 0.05$ Nsm^{-2}. We can find the sum of speed dependent components of torque losses, i.e. $T_\mu + T_h$. Thus for $\omega = 100$ rads^{-1}:

$$\begin{aligned} T_\mu + T_h &= T_{loss} - T_0 - T_f \\ T_\mu + T_h &= 22 - 3 - 5 = 14 \text{ Nm} \end{aligned}$$

From fig. 32, which shows $T_{loss} = f(\omega)$, we obtain the sum of torque losses which are independent of viscosity. For a given Δp and rotation speed ω and for $\mu = 0$ the total torque losses:

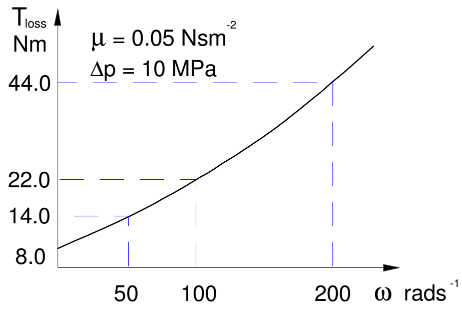

Fig. 31. Torque losses as a function of speed, $T_{loss} = f(\omega)$

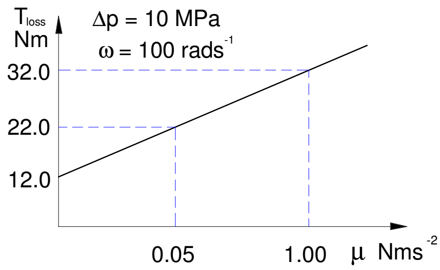

Fig. 32. Torque losses as a function of viscosity, $T_{loss} = f(\mu)$

$$T_{loss} = T_h + T_f + T_0 = 12 \text{ Nm}$$

and taking into consideration that at rotation speed $\omega = 0$ and $\Delta p = 10\,\text{MPa}$ torque losses are $T_f + T_0 = 8$ Nm (fig.30) we obtain:

$$T_h = 12 - 8 = 4 \text{ Nm} \qquad \text{answer!}$$

and:

$$T_\mu = 14 - 4 = 10 \text{ Nm} \qquad \text{answer!}$$

Finally, the values of torque losses at $\Delta p = 10$ MPa, $\omega = 10$ rads^{-1} and viscosity $\mu = 0.05$ Nsm^{-2}are:

$$T_\mu = 10 \text{ Nm},\ T_f = 5 \text{ Nm},\ T_h = 4 \text{ Nm},\ T_0 = 3 \text{ Nm} \qquad \text{(answer!)}$$